Indices

Fact (Laws of indices) —

$$a^m \times a^n = a^{m+n} \qquad a^m \div a^n = a^{m-n} \qquad (a^m)^n = a^{mn}$$

$$a^0 = 1 \qquad a^{-n} = \frac{1}{a^n} \qquad a^{\frac{1}{n}} = \sqrt[n]{a} \qquad a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$$

Example

Evaluate without a calculator:

$$81^{\frac{3}{4}} \qquad 32^{-\frac{2}{5}} \qquad \left(\frac{27}{8}\right)^{-\frac{2}{3}}$$

Example

Simplify fully:

1. $\left(32x^5\right)^{-\frac{2}{5}}$
2. $x^2\left(4x^{-\frac{1}{2}}\right)^3$
3. $\dfrac{x-4x^3}{2x^2}$, written as the sum of powers of x

Index Equations

Example

Solve:

1. $4^x = 8^{x-1}$
2. Express 9^{3x+1} in the form 3^y, giving y in terms of x.
3. $\frac{x}{\sqrt[3]{x}} = 128$

Textbook Exercises: SPS Course 1.8, Exercises 3B, 4 and 5

Surds

Fact —

$$\sqrt{ab} = \sqrt{a}\sqrt{b} \qquad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \qquad \text{but } \sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$$

To rationalise a denominator, multiply by 1 in a useful form:

$$\frac{1}{\sqrt{a}} \times \frac{\sqrt{a}}{\sqrt{a}} \qquad \frac{1}{a+\sqrt{b}} \times \frac{a-\sqrt{b}}{a-\sqrt{b}}$$

Example

Simplify:

1. $\sqrt{48} - \dfrac{6}{\sqrt{3}}$
2. $(5-\sqrt{8})(1+\sqrt{2})$

Example

Simplify $\dfrac{7+\sqrt{5}}{\sqrt{5}-1}$, giving your answer in the form $a + b\sqrt{5}$.

Example (Edexcel C1)

Solve the equation

$$10 + x\sqrt{8} = \frac{6x}{\sqrt{2}}$$

giving your answer in the form $a\sqrt{b}$ where a and b are integers.

Example

A rectangle has area $20\,\text{cm}^2$ and one side of length $(3 + \sqrt{5})$ cm. Find the length of the other side in the form $a + b\sqrt{5}$.

Textbook Exercises: SPS Course 1.7, Exercise 2 and Exam Questions 1.7.5